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Constant-Cutoff Approach to Λ (1405) Resonance in the Bound-State Soliton Model

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We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then study strong and electromagnetic properties of the $\Lambda(1405)$ hyperon in the bound-state approach to the $SU(3)$-soliton model for the hyperons, with $SU(3)$-symmetry breaking. We calculate the strong coupling constant $g_{\Lambda^* NK}$, the magnetic moment of Λ^*, the mean square radii, and the radiative decay amplitudes. Finally we compare the present results with those obtained using other models and with the available empirical data. We show that there is a general qualitative agreement between our results and the results of other models and available empirical data, except for the $\Lambda^*\pi\Sigma$ coupling, which, as in the case of the complete Skyrme model, vanishes in the second-order approximation of the kaon fluctuations used in this work.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ \tag{1.1}$$

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where

$$U = \frac{2}{F_\pi}(\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2) $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1):

$$\mathcal{L}_{\mathrm{Sk}} = \frac{1}{32e^2}\mathrm{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors [e.g., Adkins *et al.* (1983); see also Witten (1979, 1983a,b); for an extensive list of references see Holzwarth and Schwesinger (1986) and Nyman and Riska (1990)] that, after collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated the possibility of building a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. They showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4}F_\pi^2\frac{1}{\chi''(0)}a + \frac{1}{2}\frac{[\chi''(0)]^3}{(\pi/4)F_\pi^2 b}J(J + 1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4} s^2 \left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4} \mathcal{F}\right)\right] ds \tag{1.6}$$

$$b = \int_0^\infty ds \frac{64}{3} s^2 \sin^2\left(\frac{1}{4} \mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \frac{1}{4} \mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2}\left(\frac{\pi}{4}\right)^2 \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by MW is not correct since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy} \cos\left[\frac{\sqrt{7}}{2} \ln(cy) + \alpha\right], \qquad m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen adequately. When $F(0) = -n\pi$ then we want to know which of these two solutions are approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, we find that (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\, dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\, dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a,b, 1992) I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4), as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to study strong and electromagnetic properties of the $\Lambda(1405)$ hyperon in the bound-state approach to the $SU(3)$-soliton model for the hyperons, with $SU(3)$-symmetry breaking. Thus we calculate the strong coupling constant $g_{\Lambda^* NK}$, the magnetic moment of Λ^*, the mean square radii, and the radiative decay amplitudes. Finally we compare the present results with those obtained using the complete Skyrme model (Schat *et al.*, 1995), quark model (QM) (Darewych *et al.*, 1983), MIT bag model (BM) (Kaxiras *et al.*, 1985), and cloudy bag model (CBM) (Umino and Myhrer, 1991) and with the available empirical analysis of kaonic atom decays (KA) (Burkhardt and Lowe, 1991). We show that there is a general qualitative agreement between our results and the results of other models and available empirical data, except for the $\Lambda^*\pi\Sigma$ coupling, which, as in the case of the complete Skyrme model, vanishes in the second-order approximation of the kaon fluctuations used in this work.

The reason why the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ϵ is introduced, the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. Balakrishna *et al.* (1991) discussed an interesting analogy with the damped pendulum, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by equation (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been discussed in Dalarsson (1993, 1995a–c).

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into (1.1), we obtain for the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced, following Balakrishna *et al.* (1991), as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr}\right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3\mathbf{r} \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t)U_0(\mathbf{r}, t)A^+(t), \qquad U^+(\mathbf{r}, t) = A(t)U_0^+(\mathbf{r}, t)A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\boldsymbol{\tau}\cdot\mathbf{r}_0 F[r; \epsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+) \, d^3\mathbf{r} = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U \, \partial_0 U^+) \, d^3\mathbf{r} = bx^2 \, \mathrm{Tr}[\partial_0 A \, \partial_0 A^+] + c[\dot{x}(t)]^2 \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F \, y^2 \, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2 \, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy}\right)^2 + 2\sin^2 F\right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + {}^2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A \, \partial_0 A^+) = 2\dot{\alpha}_\nu\dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}\left[\frac{3}{2} \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$ and $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion-decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving exact agreement with the empirical nucleon mass.

Finally it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} & \text{for } F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for } F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. THE $SU(3)$-EXTENDED CONSTANT-CUTOFF MODEL

3.1. The Effective Interaction

The Lagrangian density for the bound-state model of hyperons is given by, with Skyrme stabilizing term omitted (Dalarasson, 1993, 1995a–c; Callan and Klebanov, 1985; Callan *et al.*, 1988),

$$\begin{aligned}\mathcal{L} &= \frac{F_\pi^2}{16}\operatorname{Tr}\partial_\mu U\,\partial^\mu U^+ + \frac{F_\pi^2}{16}m_\pi^2\operatorname{Tr}(U + U^+ - 2)\\ &\quad - \frac{1}{48}(F_K^2 - F_\pi^2)\operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U\,\partial_\mu U\,\partial^\mu U^+ + \partial_\mu U\,\partial^\mu U^+ U^+)\\ &\quad + \frac{1}{24}(F_K^2 m_K^2 - F_\pi^2 m_\pi^2)\operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U + U^+ - 2)\end{aligned} \tag{3.1}$$

where m_π and m_K are pion and kaon masses, respectively, and F_K is the kaon week-decay constant with the empirical ratio to pion decay constant $F_K/F_\pi \approx 1.23$. The first term in (3.1) is the usual σ-model Lagrangian, while the remaining three terms are all chiral- and flavor-symmetry-breaking terms, present in the mesonic sector of the model. All flavor-symmetry-breaking terms in the effective Lagrangian (3.1) also break the chiral symmetry, just as quark-mass terms do in the underlying QCD Lagrangian. In addition to the action obtained using the Lagrangian (3.1), the Wess–Zumino action in the form

$$S = -\frac{iN_c}{240\pi^2}\int d^5x\, e^{\mu\nu\alpha\beta\gamma}\operatorname{Tr}[U^+\partial_\mu U\, U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U\, U^+\partial_\gamma U] \tag{3.2}$$

must be included in the total action of a dibaryon system, where N_C is the number of colors in the underlying QCD. The Wess–Zumino action defines the topological properties of the model important for the quantization of the solitons. In the $SU(2)$ case the Wess–Zumino action vanishes identically and was therefore not present in the discussions of Sections 1 and 2.

In the present approach the meson-soliton field is written in the form

$$U = \sqrt{U_\pi}\, U_K \sqrt{U_\pi} \tag{3.3}$$

where U_π is an $SU(3)$ extension of the usual $SU(2)$ skyrmion field used to describe the nucleon spectrum, and U_K is the field describing the kaons

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp\left\{ i \frac{2^{3/2}}{F_\pi} \begin{bmatrix} 0 & K \\ K^+ & 0 \end{bmatrix} \right\} \tag{3.4}$$

In (3.4) u_π is the usual $SU(2)$-skyrmion field given by (1.4). The two-dimensional vector K in (3.5) is the kaon doublet

$$K = \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}, \qquad K^+ = [K^- \quad \overline{K}^0] \tag{3.5}$$

We now substitute (3.3), with U_π and U_K defined by (3.4), into the total action of the kaon-soliton system and expand U_K to second order in kaon fields (3.5), to obtain the effective interaction-Lagrangian density for the kaon-soliton system,

$$\begin{aligned} \mathcal{L} = {} & \dot{K}^+\dot{K} + K^+\nabla^2 K + i\lambda(r)(K^+\dot{K} - \dot{K}^+K) - m_K^2 K^+K \\ & - K^+ \cdot 2\,\frac{1 - \cos F}{r^2}\,\mathbf{I}\cdot\mathbf{L}K + K^+ v_0(r)K \end{aligned} \tag{3.6}$$

where $\mathbf{L}$ is the kaon orbital momentum and $\mathbf{I}$ is the total angular momentum of the rotating soliton. The term proportional to $\mathbf{I}\cdot\mathbf{L}$ represents the kaon-soliton (iso)spin–orbit interaction. In (3.6) we introduced the quantities $\lambda(r)$ and $v_0(r)$ as follows:

$$\lambda(r) = -\frac{N_c}{2\pi^2 F_K^2}\,\frac{\sin^2 F}{r^2}\,\frac{dF}{dr} \tag{3.7}$$

$$v_0 = \frac{1}{4}\left(\frac{dF}{dr}\right)^2 + \frac{\cos F\,(1 - \cos F)}{r^2} + \frac{F_\pi^2 m_\pi^2}{2F_K^2}(1 - \cos F) \tag{3.8}$$

The Hamiltonian density corresponding to the Lagrangian density (3.6) is given by

$$\mathcal{H} = \Pi^+\dot{K} + \dot{K}^+\Pi - \mathcal{L} \tag{3.9}$$

where

$$\Pi^+ = \partial\mathcal{L}/\partial\dot{K} = \dot{K}^+ + i\lambda(r)K^+ \tag{3.10}$$

$$\Pi = \partial\mathcal{L}/\partial\dot{K}^+ = \dot{K} - i\lambda(r)K \tag{3.11}$$

Substituting (3.6), (3.10), and (3.11) into (3.9), we obtain

$$\begin{aligned} \mathcal{H} = {} & \Pi^+\Pi - K^+\,\nabla^2 K - i\lambda(r)(K^+\Pi - \Pi^+K) + m_K^2 K^+K \\ & + K^+\left[2\,\frac{1 - \cos F}{r^2}\,\mathbf{I}\cdot\mathbf{L} - v_0(r)\right]K + \lambda^2(r)K^+K \end{aligned} \tag{3.12}$$

The kaon field (3.5) may be decomposed into modes with strangeness number $S = \pm 1$ as (Callan and Klebanov, 1985; Callan *et al.*, 1988):

$$K = \sum_m [\overline{K}_m(\mathbf{r})e^{i\omega_m t}\hat{b}_m^+ + K_m(\mathbf{r})e^{-i\omega_m t}\hat{a}_m] \tag{3.13}$$

with $\hat{a}_m$ and $\hat{b}_m^+$ annihilation and creation operators for $S = -1$ and $S = +1$ modes, respectively. From (3.12) we obtain the wave equation for the $S = -1$ mode wave functions $K_m(r)$

$$\nabla^2 K_m(\mathbf{r}) + \left[v_0(r) - 2\frac{1 - \cos F}{r^2}\mathbf{I}\cdot\mathbf{L}\right]K_m(\mathbf{r}) - m_K^2 K_m(\mathbf{r})$$
$$+ 2\omega_m\lambda(r)K_m(\mathbf{r}) + \omega_m^2 K_m(\mathbf{r}) = 0 \tag{3.14}$$

where the commutation rules for creation and annihilation operators in (3.13) give the orthonormality condition for wave functions K_m in the form

$$\int d^3\mathbf{r}\,[\omega_m + \omega_n + 2\lambda(r)]K_n^* K_m = \delta_{mn} \tag{3.15}$$

3.2. Kaon Wave Functions

Expanding the kaon wave functions $K_m(\mathbf{r})$ in terms of vector spherical harmonics (Callan and Klebanov, 1985; Callan *et al.*, 1988)

$$K(\mathbf{r}) = \sum_{\alpha,L} k_{\alpha L}(r)Y_{\alpha IL} \tag{3.16}$$

we find that the wave equation (3.14) becomes a one-dimensional differential equation. The form of interaction makes the P-state ($\alpha = 1/2$, $L = 1$) the lowest bound state representing $\Lambda(1116)$. The next state is the S-state ($\alpha = 1/2$, $L = 0$), which represents $\Lambda(1405)$. The centrifugal repulsion at short distances in the Hamiltonian density (3.12) is canceled when $\langle\mathbf{I}\cdot\mathbf{L}\rangle = -1$, i.e., for the P-state representing the $\Lambda(1405)$ hyperon. For S-wave kaons there is no cancellation and their energy is considerably higher than that of P-wave kaons. This S-wave kaon state is thus interpreted as the $\Lambda(1405)$ resonance. The binding force is obtained by leaving out $\mathbf{I}\cdot\mathbf{L}$ and $\mathbf{L}^2$ terms and it is almost entirely the Wess–Zumino force. The S-state wave function $u_0(r) = rk_{1/2,0}(r) = rk_0(r)$ satisfies the equation

$$\frac{d^2u_0}{dr^2} + v_0(r)u_0 + [\omega_0^2 - m_K^2 + 2\omega_0\lambda(r)]u_0 = 0 \tag{3.17}$$

where ω_0 is the bound-state energy. By itself the interaction $v_0(r)$ is not enough to obtain a realistic bound state, and the attractive contribution from the Wess–Zumino term must be taken into account as argued in Dalarsson (1993, 1995a–c), Callan and Klebanov (1985), and Callan *et al.* (1988).

3.3. The Rotational and Total Soliton Energies

In order to calculate the hyperon spectrum we must take into account the rotational modes of the soliton (Callan and Klebanov, 1985; Callan *et al.*, 1988). The kaon and soliton fields are rotated according to

$$\begin{aligned} K &\rightarrow a(t)K \\ U &\rightarrow A(t)UA^{+}(t) \end{aligned} \tag{3.18}$$

where

$$A(t) = \begin{bmatrix} a(t) & 0 \\ 0 & 1 \end{bmatrix} \tag{3.19}$$

is an $SU(2)$ subgroup of $SU(3)$. The $SU(2)$ rotation operator $A(t)$ adds extra time-derivative terms to the Lagrangian. Introducing the angular velocity vector $\dot{\boldsymbol{\beta}}$ using

$$(\partial_0 A^{+})A = i\dot{\boldsymbol{\beta}} \cdot \boldsymbol{\tau} \tag{3.20}$$

and the moment of inertia of the soliton $\Omega = b\epsilon^3$, where b is given by (2.8) and ϵ by (2.12), we can write the additional Lagrangian (to lowest order in the angular velocity $\dot{\boldsymbol{\beta}}$) in the following way:

$$\delta L = 2\Omega\dot{\boldsymbol{\beta}}^2 + \dot{\boldsymbol{\beta}} \cdot \int \boldsymbol{\eta}\, d^3\mathbf{r} \tag{3.21}$$

Following Dalarsson (1993, 1995a–c), we obtain the energy of the S-state, i.e., $\Lambda(1405)$ hyperon, as follows:

$$E = E_0 + \omega_0 + \frac{3}{8\Omega} Y_0 \tag{3.22}$$

where the static soliton energy E_0 is, for $m_\pi = 0$, given by (2.1) and ω_0 is the bound-state energy eigenvalue for the S state. In (3.22) Y_0 is the hyperfine splitting constant and its explicit form can easily be obtained from the general expressions found for $F_K/F_\pi \approx 1$ in Dalarsson (1993, 1995a–c) and for $F_K/F_\pi = 1.23$ in Rho *et al.* (1992).

Applying now the constant-cutoff stabilization procedure (Dalarsson, 1993, 1995a–c), we obtain for the energy of the $\Lambda(1405)$ hyperon

$$E = \omega_0 + \frac{4}{3}\left(\frac{9a^3}{8b} Y_0\right)^{1/4} \tag{3.23}$$

Table I. Numerical Results for the $\Lambda(1405)$-Hyperon Mass in MeV for $F_\pi = 186$ MeV, $F_\pi = 150$ MeV, and $F_K/F_\pi = 1.23$

Hyperon	E ($F_\pi = 186$ MeV)	E ($F_\pi = 150$ MeV)	M_{exp}
Λ^*	1433	1350	1405

The numerical results for the energy of the $\Lambda(1405)$ hyperon for $F_\pi = 186$ MeV, $F_\pi = 150$ MeV, and $F_K/F_\pi = 1.23$ are shown in Table I.

From Table I we see that, as in the case of the $\Lambda(1116)$ hyperon (Dalarsson, 1993, 1995a–c), we have good agreement with the empirical mass of the $\Lambda(1405)$ hyperon. A better result is obtained with $F_\pi = 186$ MeV, similar to the case of the $\Lambda(1116)$ hyperon (Dalarsson, 1993, 1995a–c), but in contrast to the case of nucleons. Furthermore, our results are in slightly better agreement with the empirical value than are those obtained using the complete Skyrme model in Schat *et al.* (1995).

3.4. Strong Couplings of $\Lambda(1405)$ Hyperon

The strong coupling of $\Lambda(1405)$ to the KN channel is dominant in the analysis of processes like $K^-p \to \Lambda\gamma$ and $K^-p \to \Sigma^0\gamma$. The coupling constant g_{Λ^*Kp} can be calculated using the constant-cutoff approach to the bound-state soliton model, by projection of the soliton and kaon degrees of freedom onto states with proper (iso)spin. Following the general approach of Gobbi *et al.* (1992) and using the projection identities

$$\langle \Lambda^* | A^+ | NK \rangle = \frac{-i}{(8\pi)^{1/2}} \langle \Lambda^* | -I | NK \rangle \tag{3.24}$$

$$\langle \Lambda^* | \boldsymbol{\tau} A^+ | NK \rangle = \frac{-i}{(8\pi)^{1/2}} \langle \Lambda^* | -\boldsymbol{\sigma} | NK \rangle \tag{3.25}$$

we obtain the following result:

$$\begin{aligned}
\frac{g_{\Lambda^*KN}}{(4\pi)^{1/2}} &= \frac{1}{\sqrt{2}} \int_\epsilon^\infty r^2\, dr\, k_0^2(r)[m_K\omega_0 + \lambda(r)(m_K + \omega_0) - m_K^2 - v_0(r)] \\
&= \frac{1}{\sqrt{2}} \left(\frac{9}{8ab} Y_0\right)^{3/4} \int_1^\infty y^2\, dy\, k_0^2(y) \left[m_K\omega_0 - m_K^2 + \frac{F_\pi^2 m_\pi^2}{2F_K^2}(1 - \cos F)\right] \\
&\quad + \frac{1}{\sqrt{2}} \left(\frac{9}{8ab} Y_0\right)^{1/4} \int_1^\infty dy\, k_0^2(y) \left[\frac{1}{4} y^2 \left(\frac{dF}{dy}\right)^2 + \cos F\,(1 - \cos F)\right] \\
&\quad - \frac{N_c}{2\sqrt{2}\pi^2 F_K^2}(m_K + \omega_0) \int_1^\infty dy\, k_0^2(y) \sin^2 F \frac{dF}{dy}
\end{aligned} \tag{3.26}$$

where a and b are given by (2.9) and (2.8), respectively. As in Schat *et al.* (1995), we have assumed the pseudoscalar coupling in the reduction of the interaction Lagrangian to the nonrelativistic form. However, if only the nonrelativistic form of the interaction Lagrangian is known as in the complete Skyrme model (Schat *et al.*, 1995) and here, there is no unique definition of g_{Λ^*KN}. The choice of the pseudovector coupling would introduce a factor of $\approx$0.94, which is not essential for the present qualitative investigation. Numerical calculation of equation (3.26) gives $g_{\Lambda^*KN} \approx 1.82$, which agrees rather well with the analysis of the empirical KN-scattering lengths (Lee *et al.*, 1994), where $g_{\Lambda^*KN} \approx 1.9$, and with one of the results obtained using the complete Skyrme model (CSM) in Schat *et al.* (1995), where $g_{\Lambda^*KN} \approx 1.6$, but it disagrees with a smaller value of $g_{\Lambda^*KN} \approx 0.46$ obtained using CBM in Umino and Myhrer (1991). Other $\Lambda(1405)$ strong couplings are related to the $\Lambda^*\Sigma\pi$ vertex, but, as in Schat *et al.* (1995), at least up to $O(N_c^0)$, $g_{\Lambda^*\Sigma\pi}$ vanishes. As argued in Schat *et al.* (1995), this is due to the assumption that the symmetry breaking along the strangeness direction is very strong, which is an argument to keep only the quadratic terms in the kaon-field fluctuations.

3.5. Magnetic Moments, Magnetic and Electric Radii

The electromagnetic properties of the $\Lambda(1405)$ hyperon are derived entirely from the electromagnetic current J_μ, which is obtained from the vector current $V_{a\mu}$ ($a = 1, \ldots, 8$) as follows:

$$J_\mu^{\mathrm{em}} = V_{3\mu} + 3^{-1/2} V_{8\mu} \tag{3.27}$$

The vector current $V_{a\mu}$ is obtained as the Noether current associated with the symmetry of the total action with respect to the transformation

$$U \to \exp(\tfrac{1}{2} i\epsilon^a \lambda_a)\, U \exp(-\tfrac{1}{2} i\epsilon^a \lambda_a) \tag{3.28}$$

where ϵ^a ($a = 1, \ldots, 8$) is the set of eight infinitesimally small Noether parameters. As $\epsilon^a \to 0$, we obtain from (3.28)

$$U \to U + i\epsilon^a[\lambda_a/2, U] = U + \epsilon^a\, \delta U_a \tag{3.29}$$

where $[A, B] = AB - BA$. The Noether current associated with the transformation (3.29) is

$$V_{a\mu} = 2\,\mathrm{Tr}\left[\frac{\delta S}{\delta(\partial^\mu U)}\,\delta U_a\right] = 2i\,\mathrm{Tr}\left[\frac{\delta S}{\delta(\partial^\mu U)}\left[\frac{\lambda_a}{2}, U\right]\right] \tag{3.30}$$

where S is the total action of the model. Thus we obtain

$$V_{a\mu} = -i\,\frac{F_\pi^2}{16}\,\mathrm{Tr}(\lambda_a U^+\,\partial_\mu U + \lambda_a U\,\partial_\mu U^+)$$
$$+ \frac{N_c}{96\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}(\lambda_a U^+\,\partial^\nu U U^+\,\partial^\rho U U^+\,\partial^\sigma U$$
$$- \lambda_a U\,\partial^\nu U^+ U\,\partial^\rho U^+ U\,\partial^\sigma U^+) \tag{3.31}$$

Substituting (3.31) into (3.27), we obtain the expression for the electromagnetic current. The spatial part of the electromagnetic current defined by (3.27) with (3.31) can be written as the sum of a transverse and a longitudinal component with respect to the direction $\mathbf{r}_0 = \mathbf{r}/r$. The magnetic moment, on the other hand, is defined by the expression

$$\boldsymbol{\mu} = \frac{1}{2}\int d^3\mathbf{r}\;\mathbf{r}\times\mathbf{J}^{\mathrm{em}} = \boldsymbol{\mu}_{\mathrm{O}} + \boldsymbol{\mu}_{\mathrm{I}} \tag{3.32}$$

and it is determined by the transverse component of the electromagnetic current only (Dalarsson, 1993, 1995a–c). In (3.32) $\boldsymbol{\mu}_{\mathrm{O}}$ and $\boldsymbol{\mu}_{\mathrm{I}}$ are the isoscalar and isovector contributions to the magnetic moment, and since the $\Lambda(1405)$ hyperon is an isoscalar resonance, only the term quadratic in the kaon fields in $\boldsymbol{\mu}_{\mathrm{O}}$ contributes. The explicit results for $\boldsymbol{\mu}_{\mathrm{O}}$ and $\boldsymbol{\mu}_{\mathrm{I}}$ can be found in Dalarsson (1993, 1995a–c), and following closely the procedure used there, we obtain the expression for the Λ^* magnetic moment in the form

$$\mu_{\Lambda^*} = \mu_4 = c_s\mu_1 - \frac{4}{3}\,M_N F_\pi^2 \int_\epsilon^\infty dr\; r^2 k_0^2(r)\,\sin^2\frac{F}{2}$$
$$= c_s\mu_1 - \frac{4}{3}\,M_N F_\pi^2\left(\frac{9}{8ab}\,Y_0\right)^{3/4}\int_1^\infty dy\; y^2 k_0^2(y)\,\sin^2\frac{F}{2} \tag{3.33}$$

where

$$\mu_1 = -\frac{2M_N}{3\pi\Omega}\int_\epsilon^\infty dr\; r^2\,\sin^2 F\,\frac{dF}{dr}$$
$$= -\frac{2M_N}{3\pi\Omega}\left(\frac{9}{8ab}\,Y_0\right)^{1/2}\int_1^\infty dy\; y^2\,\sin^2 F\,\frac{dF}{dy} \tag{3.34}$$

$$c_s = 1 - \frac{8}{3}\,\omega_0 \int_\epsilon^\infty dr\; r^2 k_0^2(r)\,\sin^2\frac{F}{2}$$
$$= 1 - \frac{8}{3}\,\omega_0\left(\frac{9}{8ab}\,Y_0\right)^{3/4}\int_1^\infty dy\; y^2 k_0^2(y)\,\sin^2\frac{F}{2} \tag{3.35}$$

In Dalarsson (1993, 1995a–c) it was seen that μ_1 and μ_4 are proportional to $\mu_u + \mu_d$ and μ_s, respectively, where μ_u, μ_d, and μ_s are magnetic moments of u-, d-, and s-quarks, respectively. The magnetic mean-square radius is then defined by

$$\begin{aligned}\langle r_M^2 \rangle_{\Lambda^*} &= -\frac{M_N}{5\pi\mu_{\Lambda^*}\Omega}\left(c_s \int_\epsilon^\infty r^4\,dr \sin^2 F \frac{dF}{dr}\right.\\ &\quad \left. + 2\pi\Omega \int_\epsilon^\infty r^4\,dr\,k_0^2(r) \sin^2\frac{F}{2}\right)\\ &= -\frac{M_N}{5\pi\mu_{\Lambda^*}b}\left[c_s\left(\frac{9}{8ab}Y_0\right)^{1/4}\int_1^\infty y^4\,dy \sin^2 F \frac{dF}{dy}\right.\\ &\quad \left. + 2\pi b\left(\frac{9}{8ab}Y_0\right)^{5/4}\int_1^\infty y^4\,dy\,k_0^2(y)\sin^2\frac{F}{2}\right]\end{aligned} \tag{3.36}$$

where the factor 1/5 instead of 1/3, as in (3.33) and (3.34), comes from the normalization of the magnetic form factor in the limit of zero momentum transfer (Schat *et al.*, 1995). The electric mean-square radius is given by

$$\begin{aligned}\langle r_E^2 \rangle_{\Lambda^*} &= -\frac{1}{\pi}\left\{\int_\epsilon^\infty r^4\,dr \sin^2 F \frac{dF}{dr}\right.\\ &\quad \left. - \pi \int_\epsilon^\infty r^4\,dr\,k_0^2(r)[\omega_0 + \lambda(r)]\right\}\\ &= -\frac{1}{\pi}\left(\frac{9}{8ab}Y_0\right)\int_1^\infty y^4\,dy \sin^2 F \frac{dF}{dy}\\ &\quad + \left(\frac{9}{8ab}Y_0\right)^{5/4}\int_1^\infty y^4\,dy\,k_0^2(y)[\omega_0 + \lambda(y)]\end{aligned} \tag{3.37}$$

The numerical results for magnetic moments, magnetic mean-square radii, and electric mean-square radii are composed to those obtained using the complete Skyrme model (CSM) (Schat *et al.*, 1995) in Table II.

From Table II we see that there is a general qualitative agreement between our results and those obtained using the complete Skyrme model in Schat *et al.* (1995). A detailed comparison of the predictions obtained for the Λ(1405) hyperon with those obtained for the Λ(1116) hyperon is given in Schat *et al.* (1995) and is generally valid even in the constant-cutoff approach here.

Table II. Magnetic Moments (in units of μ_p) and the Electric and Magnetic Mean-Square Radii (in fm^2) of $\Lambda(1405)$ Hyperon

	Present results	CSM results[a] Set 1	Set 2
μ/μ_p	0.06	0.08	0.09
$\langle r_M^2 \rangle$	0.95	1.14	1.21
$\langle r_E^2 \rangle$	−0.07	−0.09	−0.12

[a]Schat *et al.* (1995).

3.6. Radiative Decay Amplitudes

The $\Lambda(1405)$ hyperon has two radiative decay modes, $\Lambda^* \to \Lambda\gamma$ and $\Lambda^* \to \Sigma^0\gamma$, which are related to the isoscalar and isovector parts of the electromagnetic current, respectively. The decay amplitude for these processes is of the form

$$\Gamma = k \sum_{J_i^3, J_f^3} \sum_{\lambda=\pm 1} |\langle J_f, J_f^3 | \boldsymbol{\epsilon}_\lambda^*(\hat{\mathbf{k}}) \cdot \mathbf{J}(\mathbf{k}) | J_f, J_f^3 \rangle|^2 \tag{3.38}$$

where $k = |\mathbf{k}|$ is the magnitude of the emitted-photon momentum $\mathbf{k}$, $\hat{\mathbf{k}} = \mathbf{k}/k$ is the unit vector in the direction of the momentum $\mathbf{k}$, $\boldsymbol{\epsilon}_\lambda^*(\mathbf{k})$ is the emitted-photon polarization tensor, and $\mathbf{J}(\mathbf{k})$ is the Fourier transform of the electromagnetic current $\mathbf{J}(\mathbf{r})$, given by

$$\mathbf{J}^{\Lambda^* H}(\mathbf{k}) = i[\gamma_1^{\Lambda^* H}(k)\mathbf{T} + \gamma_2^{\Lambda^* H}(k)\mathbf{T}\cdot\hat{\mathbf{k}}\,\hat{\mathbf{k}}] \tag{3.39}$$

where $H = \{\Lambda, \Sigma^0\}$ and

$$\gamma_1^{\Lambda^* H}(k) = \int_\epsilon^\infty r^2\, dr \left\{ g_1^{\Lambda^* H}(r) j_0(kr) + \frac{1}{3} g_2^{\Lambda^* H}(r)[j_0(kr) + j_2(kr)] \right\} \tag{3.40}$$

$$\gamma_2^{\Lambda^* H}(k) = \int_\epsilon^\infty r^2\, dr\, g_2^{\Lambda^* H}(r) j_2(kr) \tag{3.41}$$

with j_0 and j_2 spherical Bessel functions of zeroth and second order, respectively, and

$$\epsilon = \left(\frac{9}{8ab} Y_0\right)^{1/4} \tag{3.42}$$

and

$$g_1^{\Lambda^*\Lambda}(r) = \frac{\cos F}{r} k_0 k_1, \qquad k_0 = k_{1/2,0}, \quad k_1 = k_{1/2,1} \tag{3.43}$$

$$g_2^{\Lambda^*\Lambda}(r) = -g_1^{\Lambda^*\Lambda}(r) + \left(k_0 \frac{dk_1}{dr} - \frac{dk_0}{dr} k_1\right) \tag{3.44}$$

$$\begin{aligned} g_1^{\Lambda^*\Sigma^0}(r) = {} & \frac{2\cos F - 1}{3} g_1^{\Lambda^*\Lambda}(r) \\ & - \frac{2N_c}{9\pi^2 F_K^2} \left[\frac{\sin 2F}{2r}\left(\omega_0 k_0 \frac{dk_1}{dr} - \omega_1 \frac{dk_0}{dr} k_1\right)\right. \\ & \left. + \frac{dF}{dr}\frac{k_0 k_1}{r}\left(\omega_0 \cos^2\frac{F}{2} + \omega_1 \sin^2\frac{F}{2} + \omega_0 \sin^2 F\right)\right] \end{aligned} \tag{3.45}$$

$$\begin{aligned} g_2^{\Lambda^*\Sigma^0}(r) = {} & \frac{2\cos F - 1}{3} g_2^{\Lambda^*\Lambda}(r) \\ & + \frac{2N_c}{9\pi^2 F_K^2} \left[\frac{\sin 2F}{2r}\left(\omega_0 k_0 \frac{dk_1}{dr} - \omega_1 \frac{dk_0}{dr} k_1\right)\right. \\ & \left. - \left(\frac{\sin 2F}{2r} - \frac{dF}{dr}\right)\frac{k_0 k_1}{r}\left(\omega_0 \cos^2\frac{F}{2} + \omega_1 \sin^2\frac{F}{2}\right)\right] \end{aligned} \tag{3.46}$$

Using (3.38)–(3.46), we finally obtain

$$\Gamma(\Lambda^* \to \Lambda\gamma) = k|\gamma^{\Lambda^*\Lambda}(k)|^2 \tag{3.47}$$

$$\Gamma(\Lambda^* \to \Sigma^0\gamma) = k|\gamma^{\Lambda^*\Sigma^0}(k)|^2 \tag{3.48}$$

where we follow the standard prescription (Schat *et al.*, 1995) and take k to be the energy difference between the initial and final hyperon states and we use the empirical value for this difference $k = k_{\mathrm{emp}}$. In Table III the numerical

Table III. Radiative Decay Amplitudes (in keV) for the Λ(1405) Resonance[a]

	Present results	CSM Set 1	CSM Set 2	QM	BM	CBM	KA
$\Gamma(\Lambda^* \to \Lambda\gamma)$	51	67	56	143	60	75	27 ± 8
$\Gamma(\Lambda^* \to \Sigma^0\gamma)$	37	29	29	91	18	2.4	23 ± 7 or 10 ± 4

[a] CSM, complete Skyrme model (Schat *et al.*, 1995); QM, quark model (Darewych *et al.*, 1983); BM, MIT bag model (Kaxiras *et al.*, 1985); CBM, cloudy bag model (Umino and Myhrer, 1991); KA, empirical analysis of kaonic atom decays (Burkhardt and Lowe, 1991).

results are compared with those obtained using the complete Skyrme model (CSM) (Schat *et al.*, 1995) quark model (QM) (Darewych *et al.*, 1983), MIT bag model (BM) (Kaxiras *et al.*, 1985), and cloudy bag model (CBM) (Umino and Myhrer, 1991) and with the available empirical analysis of kaonic atom decays (KA) (Burkhardt and Lowe, 1991).

From Table III we see that the present results are of the same order of magnitude as the results obtained by other means and there is a general qualitative agreement with both CSM results (Schat *et al.*, 1995) and KA results. It should, however, be noted that the KA results (Burkhardt and Lowe, 1991) are obtained using the uncertain value $g_{\Lambda^* KN} = 3.2$ for the strong coupling constant as an input parameter. Using the smaller value obtained here or in CSM (Schat *et al.*, 1995) would improve the agreement, as argued in Schat *et al.* (1995).

4. CONCLUSIONS

The $\Lambda(1405)$ resonance is one of the most poorly understood light-baryon states and in most quark-model calculations its rather low mass is not easy to describe (Schat *et al.*, 1995). In the present paper we used the constant-cutoff approach to the bound-state soliton model to study the strong and electromagnetic properties of the $\Lambda(1405)$ hyperon. We have calculated the strong coupling constant $g_{\Lambda^* NK}$, the magnetic moment of Λ^*, the mean square radii, and the radiative decay amplitudes.

Whenever possible we have compared the present results with those obtained using other models, e.g., the complete Skyrme model (Schat *et al.*, 1995), quark model (QM) (Darewych *et al.*, 1983), MIT bag model (BM) (Kaxiras *et al.*, 1985), and cloudy bag model (CBM) (Umino and Myhrer, 1991) and with the available empirical analysis of kaonic atom decays (KA) (Birkhardt and Lowe, 1991). We have shown that there is a general qualitative agreement between our results and the results of other models and available empirical data, except for the $\Lambda^* \pi \Sigma$ coupling, which, as in the case of the complete Skyrme model, vanishes in the second-order approximation of the kaon fluctuations used here.

On the other hand, the constant-cutoff approach employed in this paper offers a much simpler analytical structure of the results and less complicated calculations for all the quantities which describe the strong and electromagnetic properties of hyperons (Dalarsson, 1993, 1995a–c).

Finally, it should be noted that the empirical values for most of the calculated quantities are unfortunately difficult to determine. As argued in Schat *et al.* (1995) better empirical information about the $\Lambda(1405)$ resonance is needed in order to determine the quality of predictions of different models.

Some experiments to that effect are being prepared at several experimental facilities (Schat *et al.*, 1995, and references therein).

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